

Holography and trace anomaly: What is the fate of (brane-world) black holes?

Roberto Casadio*

Dipartimento di Fisica, Università di Bologna, and I.N.F.N., Sezione di Bologna, via Irnerio 46, 40126 Bologna, Italy

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The holographic principle relates (classical) gravitational waves in the *bulk* to quantum fluctuations and the Weyl anomaly of a conformal field theory on the boundary (the *brane*). One can thus argue that linear perturbations in the bulk of static black holes located on the brane be related to the Hawking flux and that (brane-world) black holes are therefore unstable. We try to gain some information on such instability from established knowledge of the Hawking radiation on the brane. In this context, the well-known trace anomaly is used as a measure of both the validity of the holographic picture and of the instability for several proposed static brane metrics. In light of the above analysis, we finally consider a time-dependent metric as the (approximate) representation of the late stage of evaporating black holes which is characterized by decreasing Hawking temperature, in qualitative agreement with what is required by energy conservation.

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I. INTRODUCTION

The holographic principle [1], in the form of the anti-de Sitter (AdS) conformal field theory (CFT) conjecture [2], when applied to the Randall-Sundrum (RS) brane-world [3], yields a relation between classical gravitational perturbations in the bulk and quantum fluctuations of conformal matter fields on the brane [4]. It was then proposed in Refs. [5,6] that black hole metrics which solve the bulk equations with brane boundary conditions, and whose central singularities are located on the brane, genuinely correspond to quantum corrected (semiclassical) black holes on the brane, rather than to classical ones. A major consequence of such a conjecture would be that no static black holes exist in the brane-world [7], since semiclassical corrections (approximately described by a conformal field theory (CFT) on the brane) have the form of a flux of energy from the source. That black holes are unstable was already well known in the four-dimensional theory, since the Hawking effect [8] makes such objects evaporate, and the semiclassical Einstein equations should hence contain the *back reaction* of the evaporation flux on the metric. The novelty is that, from the bulk side (of the AdS-CFT correspondence), one understands the Hawking radiation in terms of classical gravitational waves whose origin is the acceleration the black holes are subject to, living on a nongeodesic hypersurface of the (asymptotically) anti-de Sitter space-time AdS₅ [9].

In Ref. [10] a method was proposed by which static brane metrics, such as the asymptotically flat ones put forward in Refs. [11–14], can be extended into the bulk [15]. The method makes use of the multipole expansion (with respect to the usual areal radial coordinate r on the brane) and proved particularly well suited for very large black holes with

$$M \gg \sigma^{-1}, \quad (1)$$

where M is the four-dimensional Arnowitt-Deser-Misner (ADM) mass parameter (in geometrical units) and 3σ the

brane tension (times the five-dimensional gravitational constant). The main result is that the horizon of an astrophysical size black hole has the shape of a “pancake” (see also Ref. [16]) and its area is roughly equal to the four-dimensional expression (in terms of M). To the extent at which the employed approximation is reliable, no singular behavior in the curvature and Kretschmann scalars in the bulk was found, contrary to the case of the black string (BS) of Ref. [17]. Possible caveats of this approach have already been thoroughly discussed in Ref. [10]. In particular, the convergence of the multiple expansion on the axis of cylindrical symmetry which extends into the bulk is hard to test and the resulting metrics are not completely reliable thereon. As a consequence, it is hard to determine whether the bulk geometry contains singularities on the axis (while no singularity seems to occur far from it) and whether it is asymptotically AdS away from the brane near the axis (while it appears asymptotically AdS far from the axis). In fact, in recent numerical works, regular brane metrics were shown to develop singularities in the bulk when extended by a different method [18] or problems emerged when trying to describe large black holes in asymptotically AdS bulk [19].

If the conjecture of Refs. [5–7] is correct, it then follows that the static bulk solutions found in Ref. [10] have singularities on the cylindrical axis (possibly far away from the brane) or, at least, are unstable under linear perturbations (of the metric in the bulk), as it occurs for the BS [20]. In either case, it is likely that such metrics will evolve toward different (more stable but yet unknown) configurations [7]. Since the metric elements in Ref. [10] are expressed as sums of many terms (multipoles) and those in Refs. [18,19] are expressed only numerically, it is practically impossible to carry out a linear perturbation analysis on such backgrounds. One could otherwise try to use the AdS-CFT correspondence backwards in order to estimate the overall effect of bulk perturbations from the established knowledge of the Hawking radiation on the brane. Some information on the latter can be determined straightforwardly from standard four-dimensional expressions provided the brane metric is given (see, e.g., Ref. [21]). One must then check that such infor-

*Electronic address: casadio@bo.infn.it

mation is consistent with known features of the AdS-CFT correspondence before drawing any conclusion about the bulk stability.

In fact, the AdS-CFT correspondence requires some general conditions to hold. First of all, one needs the planar limit of the large \mathcal{N} expansion of the CFT, that is, a large number of conformal fields [2],

$$\mathcal{N} \sim \sigma^{-2} \ell_p^{-2} \gg 1. \quad (2)$$

where, in RS models, the four-dimensional Newton constant $8\pi G_N = \ell_p^2$ (ℓ_p being the Planck length) is related to the fundamental gravitational length ℓ_g in five dimensions by $\ell_p^2 = \sigma \ell_g^3$ [3,22]. Equation (2) is therefore tantamount to $\ell_g \gg \ell_p$ and assures that stringy effects are negligible. Moreover, the presence of the brane introduces a normalizable four-dimensional graviton and an ultraviolet (UV) cutoff in the CFT,

$$\lambda_{UV} \sim \sigma^{-1}. \quad (3)$$

The latter must of course be much shorter than any physical low energy scale λ_{IR} of interest,

$$\lambda_{UV} \ll \lambda_{IR}, \quad (4)$$

in order for the CFT description of the brane-world to be consistent. A further remark is in order about the relevance of the bulk being asymptotically AdS away from the brane. We have already mentioned that, according to Ref. [10], possible singularities in the bulk (and the corresponding deviations from asymptotic AdS) should be located on the axis. To what extent such singularities restrict the holographic description is hard to tell *a priori* and a general criterion for the validity of the AdS-CFT correspondence will therefore be given in Sec. II [see Eq. (12) and the following discussion].

So far, no practical advantage seems to emerge from the holographic description over the standard four-dimensional treatment of the back-reaction problem. There is, however, a point of attack: As we shall review shortly, in the brane-world a vacuum (brane) solution needs to satisfy just one equation, whose analysis is therefore significantly simpler than the full set of four-dimensional vacuum Einstein equations. This will allow us to attempt an approximate description of the late stage of the evaporation which is in qualitative agreement with earlier studies of black holes as extended objects in the microcanonical picture [23,24], and is characterized by a black hole temperature which vanishes along with the mass of the black hole. Let us remark that the total energy of the system (black hole plus Hawking radiation) is conserved in the microcanonical ensemble and this suggests that the final evolution of a black hole is really unitary.

In the next section we summarize the general concepts with a particular emphasis on the trace anomaly as derived from the point of view of four-dimensional quantum field theory and its comparison with that predicted by the AdS-CFT correspondence. In Sec. III, we then apply this formalism to candidate static black holes in order to check the reliability of their holographic picture and stability. Our conclusions first of all support the view given in Refs. [10,25]

that static metrics are a good approximation for astrophysical black holes. Moreover, some of the brane metrics analyzed in Refs. [10,14] are shown to allow for a closer holographic interpretation and to be more stable than the BS. This suggests that brane-world black holes might evaporate more slowly than they would do in a four-dimensional space-time already for very large ADM masses [26]. Finally, in Sec. IV we discuss a possible candidate time-dependent metric to estimate the late stage of the evaporation by self-consistently including the trace anomaly in the relevant vacuum equation.

We shall adopt the brane metric signature $(-, +, +, +)$ and units with $\hbar = c = 1$. Latin indices i, j, \dots will denote brane coordinates throughout the paper.

II. GENERAL FRAMEWORK

The five-dimensional Einstein equations in (asymptotically) AdS₅ with bulk cosmological constant Λ can be projected onto the brane by making use of the Gauss-Codazzi relations and Israel's junction conditions (see Ref. [22] for the details). For the RS case which we consider throughout the paper $\Lambda = -\sigma^2 \ell_g^3/6$ [3], so that the brane cosmological constant vanishes and the effective four-dimensional Einstein equations become

$$G_{ij} = \ell_p^2 \tau_{ij} + \frac{\ell_p^4}{\sigma^2} \pi_{ij} + E_{ij}, \quad (5)$$

where $G_{ij} = R_{ij} - (1/2)Rg_{ij}$ is the four-dimensional Einstein tensor, τ_{ij} is the energy-momentum tensor of matter localized on the brane (there is no matter in the bulk) and

$$\pi_{ij} = -\frac{1}{4} \tau_{ik} \tau_j^k + \frac{1}{12} \tau \tau_{ij} + \frac{1}{8} g_{ij} \tau_{kl} \tau^{kl} - \frac{1}{24} g_{ij} \tau^2. \quad (6)$$

Where no matter appears on the brane ($\tau_{ij} = 0$), the existence of an extra spatial dimension manifests itself in the brane-world only in the form of the nonlocal source term E_{ij} , which is (minus) the projection of the bulk Weyl tensor on the brane and must be traceless [5,22]. Vacuum brane metrics therefore satisfy

$$R_{ij} = E_{ij} \quad (7)$$

$$\Downarrow$$

$$R = 0. \quad (8)$$

Of course, Eq. (8) is a weaker condition than the four-dimensional vacuum equation $R_{ij} = 0$ and, consequently, Birkhoff's theorem does not necessarily hold for spherically symmetric vacuum brane metrics.

The AdS-CFT correspondence should relate the tensor E_{ij} representing (classical) gravitational waves in the bulk to the expectation value of the (renormalized) energy-momentum tensor of conformal fields on the brane [4]. Let us denote the latter by $\langle T_{ij} \rangle$. One should then have

$$E_{ij}^i \sim \ell_p^2 \langle T_{ij}^i \rangle. \quad (9)$$

Since the left hand side above is traceless, such a correspondence can hold if $\langle T \rangle \equiv \langle T^i_i \rangle = 0$, that is, if the conformal symmetry is not anomalous. Of course, this requires that the UV cutoff (3) be much shorter than any physical length scale in the system. It also requires a “flat” brane (i.e., the absence of any intrinsic four-dimensional length associated with the background) otherwise the CFT will also be affected by that scale. For a black hole, the gravitational radius $r_h \sim M$ is a natural length scale and one therefore expects that only CFT modes with wavelengths much shorter than r_h [27] (and still much larger than σ^{-1}) propagate freely. One then finds that the necessary condition (4) is equivalent to Eq. (1), that is a reliable CFT description of the Hawking radiation might be possible only for very large black holes of the kind considered in Ref. [10].

From the point of view of the AdS-CFT correspondence, it is the value of bulk perturbations at the boundary that acts as a source for the CFT fields and can give rise to $\langle T \rangle_{\text{CFT}}$. As a check, one can compare with the trace anomaly induced by the presence of a brane as a boundary of AdS in several theories in which the AdS-CFT applies. Since we are just interested in a four-dimensional brane, the case of relevance to us is that of (\mathcal{N} stacked) D3-branes (possibly with $R = 0$) embedded in AdS_5 . For such a configuration one finds the holographic Weyl anomaly [28]

$$\langle T \rangle_{\text{CFT}} = \frac{1}{4\ell_p^2 \sigma^2} \left(R_{ij} R^{ij} - \frac{1}{3} R^2 \right), \quad (10)$$

which reproduces the conformal anomaly of the four-dimensional $\mathcal{M}=4$ superconformal $SU(\mathcal{N})$ gauge theory in the large \mathcal{N} limit (2) and vanishes in a four-dimensional (Ricci flat) vacuum [29].

On the other hand, the trace anomaly of the pertinent four-dimensional field theory, $\langle T \rangle_{4D} = \langle T^i_i \rangle$, can be evaluated independently. It is given in terms of geometrical quantities as well and numerical coefficients which depend on the matter fields. Further, it does not usually vanish on a curved background (even if it is Ricci flat) because, contrary to $\langle T \rangle_{\text{CFT}}$, it also contains the Kretschmann scalar $R_{ijkl} R^{ijkl}$. For example, one finds for n_B boson fields (see, e.g., Ref. [21])

$$\langle T \rangle_{4D} = \frac{n_B}{2880\pi^2} (R_{ijkl} R^{ijkl} - R_{ij} R^{ij} - \square R). \quad (11)$$

The term $\square R$, which is renormalization dependent, would, however, vanish according to Eq. (8) but we include such term for future reference (see, in particular, Sec. IV). It is this nonvanishing trace $\langle T \rangle_{4D}$ which measures the actual violation of the conformal symmetry on the brane.

If $\langle T \rangle_{4D} \neq \langle T \rangle_{\text{CFT}}$, one needs more than the AdS-CFT correspondence to describe the brane physics for the chosen background. In other words, this inequality can be interpreted as signaling the excitation of other matter fields living on the brane (with $\tau \equiv \tau^i_i \sim \langle T \rangle_{4D} - \langle T \rangle_{\text{CFT}}$). The relative difference with respect to $\langle T \rangle_{\text{CFT}}$,

$$\Gamma_{\text{CFT}} \equiv \left| \frac{\langle T \rangle_{4D} - \langle T \rangle_{\text{CFT}}}{\langle T \rangle_{\text{CFT}}} \right|, \quad (12)$$

can then be used to estimate to what extent classical gravitational waves in the bulk determine matter fluctuations on the brane [30]. If $\Gamma_{\text{CFT}} \ll 1$, then the AdS-CFT conjecture implies that the (quantum) brane and (classical) bulk descriptions of black holes are equivalent. Otherwise, since the holography can just account for that part of the Hawking flux which is responsible for $\langle T \rangle_{\text{CFT}}$, the ratio Γ_{CFT} is also a measure of the relative strength of brane fluctuations (involving other matter modes) with respect to bulk gravitational waves.

From the four-dimensional point of view, the trace anomaly is evidence that one is quantizing matter fields, by means of the background field method, on the “wrong” (i.e., unstable) background metric. One should instead find a background and matter state (both necessarily time dependent) whose metric and energy-momentum tensor solve all relevant field equations simultaneously. This is the aforementioned back-reaction problem of Hawking radiation, whose solution is still out of grasp after several decades from the discovery of black hole evaporation. The authors of Ref. [6] argue that, because of the AdS-CFT correspondence, the problem of describing a brane-world black hole is just as difficult as the (four-dimensional) back-reaction problem itself. One could go even further and claim that it is *at least* as difficult, since for an holographic black hole the AdS-CFT should be exact and $\langle T \rangle_{4D} = \langle T \rangle_{\text{CFT}}$ (i.e., $\Gamma_{\text{CFT}} = 0$), but realistic black holes might involve more “ingredients.” If, however, one focuses on the brane description, and just considers Eq. (8), the task will simplify considerably.

III. STATIC BLACK HOLES?

We first want to analyze both the semiclassical stability and holography of candidate static brane-world black holes. They are described by asymptotically flat, spherically symmetric metrics which solve Eq. (8), and can be put in the form

$$ds^2 = -N(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2, \quad (13)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and for the functions N and A we shall now consider several cases previously appeared in the literature.

A. 4D and CFT trace anomalies

The first step is to compute $\langle T^i_j \rangle$, $\langle T \rangle_{4D}$ and $\langle T \rangle_{\text{CFT}}$. The ratio Γ_{CFT} will then give a measure of the reliability of the holographic picture.

1. Black string

We take the Schwarzschild line element (corresponding to the BS [17]),

$$N = \frac{1}{A} = 1 - \frac{2M}{r}, \quad (14)$$

as the reference brane metric, whose horizon radius is $r_h = 2M$. The trace of the energy-momentum tensor for n_B boson fields on this background is

$${}^S\langle T \rangle_{4D} = \frac{n_B M^2}{60\pi^2 r^6}, \quad (15)$$

and four-dimensional covariant conservation relates it to those parts of the diagonal components of the energy-momentum tensor which do not depend on the quantum state of the radiation [31]. Moreover, in the Unruh vacuum, one also has the flux of outgoing Hawking radiation [21,31],

$${}^S\langle T^t_t \rangle = {}^S\langle T^r_r \rangle = -{}^S\langle T^r_t \rangle = -{}^S\langle T^t_r \rangle \sim \frac{n_B K}{M^2 r^2}, \quad (16)$$

where K is a dimensionless constant. It is therefore expected that the components of the Ricci tensor of the metric which incorporates the back-reaction of the Hawking radiation have a leading behavior

$${}^S E^i_j \sim \frac{n_B \ell_p^2}{M^2 r^2}. \quad (17)$$

The above terms, representing a steady flux of radiation, are consistent within the adiabatic approximation for which the background (brane) metric is held static. In fact, the constant K (formally) arises from an integration performed over an infinite interval of (Euclidean) time, so as to include all the poles in the Wightman function of the radiation field which yield the Planckian spectrum (the integral diverges and is divided by the length of the time interval to estimate the average flux per unit time [8]). During this large time, the change in the ADM mass is neglected. In order to solve for the back reaction, one should instead consider the full time dependence of the metric and black hole source.

From the point of view of the bulk space-time, the Hawking flux just modifies the junction equations [32] at the brane. The latter, on account of the orbifold symmetry Z_2 , will in general read [22]

$$[K_{ij}] = \ell_g^3 \left(T_{ij} - \frac{1}{3} g_{ij} T \right). \quad (18)$$

The quantity $[K_{ij}]$ is the jump in the extrinsic curvature of the brane and T_{ij} the source term localized on the brane which, for the case at hand, contains the vacuum energy 3σ and the Hawking flux. One thus has

$$[K_{ij}] = -\sigma g_{ij} + \frac{\ell_p^2}{\sigma} \left({}^S\langle T_{ij} \rangle - \frac{1}{3} g_{ij} {}^S\langle T \rangle_{4D} \right). \quad (19)$$

Since $r > 2M$ outside the horizon, the second term in the right hand side above is negligible with respect to the first one everywhere in the space accessible to an external observer if

$$M\sigma \gg \frac{\ell_p}{M}. \quad (20)$$

This shows that the Hawking radiation just gives rise to a small perturbation of the bulk metric for black holes of astrophysical size [for which Eq. (1) holds]. This is assumed in the approach of Ref. [10] (and also in the numerical analysis of Refs. [25]) to justify staticity [33].

However, the holographic Weyl anomaly (10) vanishes for this metric (the ratio $\Gamma_{\text{CFT} \rightarrow \infty}$) since $R_{ij} = 0$, and drawing any conclusion from the AdS-CFT correspondence looks questionable.

2. Case I

From Refs. [12,14], we consider the functions

$$N = 1 - \frac{2M}{r}, \quad A = \frac{\left(1 - \frac{3M}{2r}\right)}{N \left[1 - \frac{3M}{2r} \left(1 + \frac{4}{9}\eta\right)\right]}, \quad (21)$$

the causal structure of whose geometry was analyzed in details in Ref. [14]. One finds that the nonvanishing Ricci tensor components are given by

$$\begin{aligned} {}^I R^t_t &= \frac{4\eta M^2}{3(2r-3M)^2 r^2}, \\ {}^I R^r_r &= -\frac{4\eta M}{3(2r-3M)r^2}, \\ {}^I R^\theta_\theta &= {}^I R^\phi_\phi = -\frac{4\eta M(r-M)}{3(2r-3M)^2 r^2}. \end{aligned} \quad (22)$$

Note that the corrections in Eq. (17) that one obtains from the Hawking radiation dominate (in the large r approximation) over those following from the components in Eq. (22). This is expected since the metric (21) does not contain an outgoing flux and is asymptotically flat. However, the corrections in Eq. (22) certainly overcome the Hawking flux in the interval

$$1 \ll \frac{r}{M} \lesssim |\eta| \frac{M^2}{\ell_p^2}, \quad (23)$$

whose extension can be very large for astrophysical black holes (with $M \gg \sigma^{-1} \geq \ell_p$) provided η is not infinitesimal [34]. Moreover, the trace of the energy-momentum tensor of a boson field on this four-dimensional background has a leading behavior for large r given by (see Appendix A for more details)

$${}^I\langle T \rangle_{4D} \approx \left(1 + \frac{\eta}{3} + \frac{\eta^2}{24}\right) {}^S\langle T \rangle_{4D}, \quad (24)$$

which is of the same order in $1/r$ as the trace in Eq. (15).

The expected trace anomaly from the AdS-CFT correspondence on this background is of the same order as ${}^I\langle T \rangle_{4D}$, namely

$${}^I\langle T \rangle_{\text{CFT}} \sim \frac{\eta^2 M^2}{6 \ell_p^2 \sigma^2 r^6}. \quad (25)$$

Hence, for small $|\eta|$, the ratio

$${}^I\Gamma_{\text{CFT}} \sim \left| 1 - \frac{n_B \ell_p^2 \sigma^2}{10 \eta^2 \pi^2} \right|, \quad (26)$$

which is finite for $\eta \neq 0$ and represents a significant improvement over the BS. In particular, one has that ${}^I\Gamma_{\text{CFT}} \approx 0$ for

$$\eta \approx \pm \frac{\sqrt{n_B} \ell_p \sigma}{\sqrt{10} \pi} \equiv {}^I\eta, \quad (27)$$

where we used $\ell_p \sigma \ll 1$. For $\eta \approx {}^I\eta$ one expects that the holographic principle yields a reliable description of such black holes. Note, however, that the rough estimate $n_B \sim \mathcal{N}$ from Eq. (2) would yield $|{}^I\eta| \approx 0.1$ which is significantly larger than the experimental bound $|\eta| \lesssim 10^{-4}$ from solar system measurements [35].

3. Case II

From Refs. [13,14], let us now consider the metric described by the functions (for the causal structure see again Ref. [14])

$$N = \left[\frac{\eta + \sqrt{1 - \frac{2M}{r}(1+\eta)}}{1+\eta} \right]^2, \quad (28)$$

$$A = \left[1 - \frac{2M}{r}(1+\eta) \right]^{-1},$$

which yield the non-vanishing Ricci tensor components

$${}^{\text{II}}R^r_r = -2 {}^{\text{II}}R^\theta_\theta = -2 {}^{\text{II}}R^\phi_\phi$$

$$= \frac{2\eta(1+\eta)M}{\left(\eta + \sqrt{1 - \frac{2M}{r}(1+\eta)} \right) r^3}. \quad (29)$$

These are again subleading at large r with respect to the radiation terms in Eq. (17), but of the same order as those of case I, and the estimate in Eq. (23) applies to this case as well. The trace of the boson energy-momentum tensor is also of the same order in $1/r$ as that in Eq. (15) (see Appendix A),

$${}^{\text{II}}\langle T \rangle_{4D} \approx (1+\eta) {}^S\langle T \rangle_{4D}, \quad (30)$$

and the conformal anomaly from the AdS-CFT correspondence is

$${}^{\text{II}}\langle T \rangle_{\text{CFT}} \sim {}^I\langle T \rangle_{\text{CFT}}, \quad (31)$$

yielding a finite (for $\eta \neq 0$) ratio ${}^{\text{II}}\Gamma_{\text{CFT}} \sim {}^I\Gamma_{\text{CFT}}$ and ${}^{\text{II}}\Gamma_{\text{CFT}} \approx 0$ for

$$\eta \approx {}^I\eta/3, \quad (32)$$

where we again used $\ell_p \sigma \ll 1$ and $|\eta| \ll 1$.

4. Case III

Finally, from Ref. [11], the metric

$$N = \frac{1}{A} = 1 - \frac{2M}{r} + \frac{\eta M^2}{2r^2}, \quad (33)$$

has the Ricci tensor components

$${}^{\text{III}}R^t_t = {}^{\text{III}}R^r_r = -{}^{\text{III}}R^\theta_\theta = -{}^{\text{III}}R^\phi_\phi = \frac{\eta M^2}{2r^4}. \quad (34)$$

The interval over which such corrections overcome the Hawking flux is now narrower, namely

$$1 \ll \frac{r}{M} \lesssim \sqrt{|\eta|} \frac{M}{\ell_p}, \quad (35)$$

but still quite large for astrophysical black holes. The corresponding trace anomaly is given by

$${}^{\text{III}}\langle T \rangle_{4D} \approx {}^S\langle T \rangle_{4D}, \quad (36)$$

to leading order in $1/r$ (see Appendix A).

The AdS-CFT trace anomaly is subleading for this case, namely

$${}^{\text{III}}\langle T \rangle_{\text{CFT}} \approx \frac{\eta^2 M^4}{\ell_p^2 \sigma^2 r^8}, \quad (37)$$

and

$${}^{\text{III}}\Gamma_{\text{CFT}} \sim n_B \frac{\ell_p^2 \sigma^2 r^2}{\eta^2 M^2}, \quad (38)$$

which is larger than those of cases I and II. This result seems therefore to disfavor such a metric as a candidate holographic black hole.

B. Semiclassical stability

Since the trace $\langle T \rangle_{4D} \neq 0$ signifies that the chosen background is not the true vacuum (for which one would rather expect a vanishing conformal anomaly and toward which the system will evolve), a quantitative way of estimating the stability of the above solutions with respect to the BS is to evaluate the ratio

$$\Gamma_{4D} \equiv \left| \frac{\langle T \rangle_{4D}}{{}^S\langle T \rangle} \right|. \quad (39)$$

In regions where $\Gamma_{4D} \ll 1$, the corresponding metric should be more stable than the BS. This occurs, for instance, for the candidate small black hole metrics which we shall analyze in

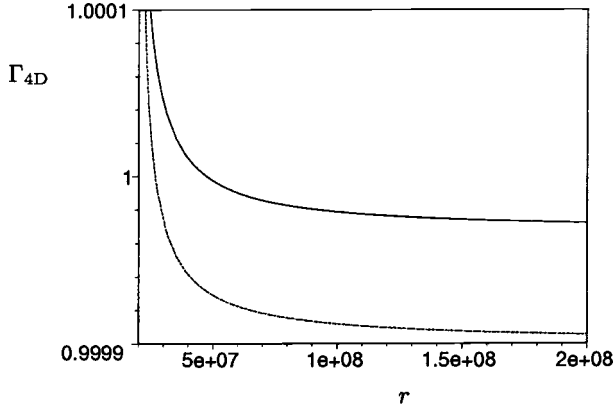


FIG. 1. The ratios $\text{I}\Gamma_{4D}$ (solid line) and $\text{II}\Gamma_{4D}$ (dashed line) for $\eta = -10^{-4}$ and $M = 10^7 \sigma^{-1}$ (r is in units of σ^{-1}).

Sec. III B 2 [see Eqs. (46)]. Such brane metrics violate the condition (4) and are therefore unlikely to admit an holographic description, as our approach will indeed confirm [36].

I. Cases I, II, and III

It is interesting to take note of the approximate asymptotic values of the ratio Γ_{4D} at large r for the three cases previously discussed:

$$\begin{aligned}\text{I}\Gamma_{4D} &\rightarrow 1 + \frac{\eta}{3} + \frac{\eta^2}{24}, \\ \text{II}\Gamma_{4D} &\rightarrow 1 + \eta, \\ \text{III}\Gamma_{4D} &\rightarrow 1.\end{aligned}\quad (40)$$

From such expressions, it appears that the preferred solutions are again given by cases I and II, but with $\eta < 0$ (in Refs. [10,14] we already discussed some reasons why one expects $\eta < 0$ and the above results further support this conclusion since $\eta > 0$ always leads to a larger value of the trace than $\eta \leq 0$). Case III instead represents no real improvement over the BS.

In detail, the ratios $\text{I}\Gamma_{4D}$ and $\text{II}\Gamma_{4D}$ are plotted in Fig. 1 for the typical values $M = 10^7 \sigma^{-1} \approx 1$ km and $\eta = -10^{-4}$ [10]. Except for the region very near the horizon ($r_h = 2M$), the ratios $\text{II}\Gamma_{4D} < \text{I}\Gamma_{4D} < 1$. This might signal a stronger instability near the horizon than for the BS which, however, becomes milder at large distances.

2. Small black holes

There are more candidate metrics for small black holes with $M\sigma \lesssim 1$ (see, e.g., Refs. [16,37,38]), namely the higher-dimensional Schwarzschild metrics [39]

$$N = \frac{1}{A} = 1 - \left(\frac{r_h}{r}\right)^n, \quad (41)$$

with $n \geq 2$. Unfortunately one cannot rigorously identify $r_h \approx 2M$, since the four-dimensional ADM mass for this background is zero [40], and confronting with the BS (corre-

sponding to $n=1$) becomes more subtle. Let us anyways assume $r_h \sim M_{(n)}$ holds from Newtonian arguments (at least for $n=2$ [37,41]), where the $M_{(n)}$'s are now understood as the multipole moments of the energy distribution of the source. One then obtains

$$\begin{aligned}{}^{(n)}R^t_t &= {}^{(n)}R^r_r = -\frac{n}{2}{}^{(n)}R^\theta_\theta = -\frac{n}{2}{}^{(n)}R^\phi_\phi \\ &= \frac{n(n-1)}{2} \frac{M_{(n)}^n}{r^{2+n}}.\end{aligned}\quad (42)$$

Note that the scalar

$${}^{(n)}R = (n-1)(n-2) \frac{M_{(n)}^n}{r^{2+n}} \quad (43)$$

just vanishes for $n=1$ (four-dimensional Schwarzschild) and $n=2$ (five-dimensional Schwarzschild). These are the only cases which satisfy the vacuum Eq. (8).

The complete expression for the trace anomaly is given in Eq. (A1d) which shows that, for $n=2$, one has

$${}^{(2)}\langle T \rangle_{4D} = n_B \frac{13M_{(2)}^4}{720\pi^2 r^8}, \quad (44)$$

while, for $n > 2$, the leading behavior at large r is given by the $\square R$ term, that is

$${}^{(n>2)}\langle T \rangle_{4D} \sim n_B \frac{(n^2-1)(n^2-4)M_{(n)}^n}{5760\pi^2 r^{4+n}}, \quad (45)$$

both of which never vanish. However, and although the numerical coefficient is questionable because of the aforementioned ambiguity in relating $M_{(n)}$ to r_h , the ratios

$$\begin{aligned}{}^{(2)}\Gamma_{4D} &\sim \left(\frac{M_{(2)}}{r}\right)^2, \\ {}^{(n>2)}\Gamma_{4D} &\sim \left(\frac{M_{(n)}}{r}\right)^{n-2}\end{aligned}\quad (46)$$

tend to zero at large r and are less than one for $n > 1$ and $r \gtrsim M_{(n)}$. This makes such metrics better candidates to describe very small black holes on the brane.

It is then interesting to study their extension in the bulk by applying the method of Ref. [10]. Details are given in Appendix B for $n=2$ (see also Ref. [42]). For $M_{(2)}\sigma = 1$ the value of ρ^2 along geodesic lines of constant r is displayed in Fig. 2 and the horizon is approximately given by the line $r = M_{(2)}$. Note that it is slightly flattened since the maximum value of z along such a line is about $0.7\sigma^{-1}$. It would, however, depart more and more from that curve the smaller $M_{(2)}\sigma$ is. For larger values of $M\sigma$, one expects a nonvanishing ADM mass ($2M = M_{(1)}$), and the line $r = r_h$ is then flatter and a better approximation of the true location for the horizon (see Ref. [10] for cases I, II, and III).

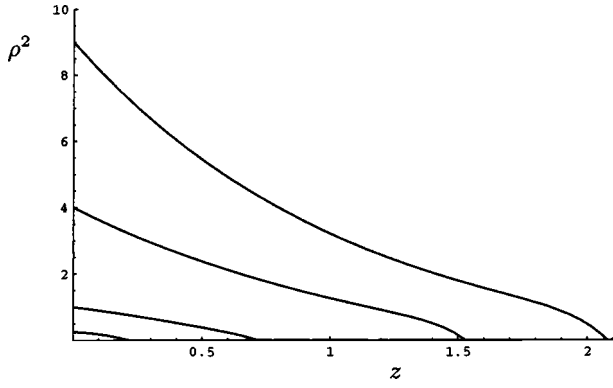


FIG. 2. The function ρ^2 for $M_{(2)} = \sigma^{-1}$ and $r = M_{(2)}/2$, $M_{(2)}$, $2M_{(2)}$, and $3M_{(2)}$. Terms up to order $1/r^{13}$ are included.

The trace anomaly from the AdS-CFT correspondence is given in Eq. (A2d), and just vanishes for $n=1$. Neglecting numerical coefficients, one thus obtains, for $n=2$, a ratio

$${}^{(2)}\Gamma_{\text{CFT}} \approx 1 - n_B \frac{13\ell_p^2 \sigma^2}{720\pi^2}, \quad (47)$$

which vanishes for

$$n_B \approx \frac{720\pi^2}{13\ell_p^2 \sigma^2} \gg 1, \quad (48)$$

where the inequality follows from the condition (2). For such a number of boson fields one has

$${}^{(2)}\langle T \rangle_{4D} \approx \frac{M_{(2)}^4}{\ell_p^2 \sigma^2 r^8}. \quad (49)$$

For $n > 2$, instead

$${}^{(n>2)}\Gamma_{\text{CFT}} \sim \left(\frac{r}{M_{(n)}} \right)^n, \quad (50)$$

which is an increasing function of n and diverges for $r \rightarrow \infty$ as occurred for case III. It thus seems that, although such brane metrics are semiclassically more stable, they significantly depart from the holographic description for increasing n . This is not contradictory, since the condition (4) [or, equivalently, Eq. (1)] is now violated and one expects that the CFT description fails. Moreover, one also expects that the smaller the black hole (horizon), the finer the space-time structure is probed, and one eventually needs to include stringy effects.

IV. TIME-DEPENDENT BLACK HOLES: A CONJECTURE

Solving Einstein equations for time-dependent metrics is in general a formidable task, unless symmetries are imposed to freeze enough degrees of freedom, and there is little hope to find analytic solutions for the present case [43]. Let us then begin with a qualitative remark based on the results we have shown in the previous section: Just looking at the trace anomaly (15) one is led to conclude that the natural time

evolution of a four-dimensional black hole is toward smaller and smaller ADM masses, since ${}^S\langle T \rangle_{4D} = 0$ for $M = 0$. However, such an evolution seems to make the black hole less and less stable, its specific heat being more and more negative and the temperature diverging, as is argued in the usual canonical picture of the Hawking radiation [8]. But the picture might change if one considers a fully dynamical description (for a recent four-dimensional analysis of Hawking evaporation and trace anomaly, see Ref. [44]).

In order to substantiate our argument, let us replace Eq. (8) with the semiclassical brane equation

$$R = -\ell_p^2 \langle T \rangle_{4D}, \quad (51)$$

and note that, for a metric of the form (41) and time-independent r_h , the scalar curvature R always falls off more slowly at large r than the corresponding trace anomaly (A1d). Hence, Eq. (51) cannot be solved by such an *ansatz*. However, the situation changes when the metric is time dependent: for an asymptotically flat brane metric, on expanding to leading order at large r , the term $\square R$ becomes of the same leading order as R and dominates in the expression of the trace anomaly [45]. In particular, Eq. (51) becomes

$$R \sim n_B \ell_p^2 \ddot{R}, \quad (52)$$

where a dot denotes the derivative with respect to t . We then reconsider the Schwarzschild metric (14) with the simple expression for the ADM mass,

$$M = M_0 e^{-at}, \quad (53)$$

where $a > 0$ so as to enforce decreasing mass. The Ricci scalar and the trace anomaly for this metric, to leading order at large r , are given by

$$R = 2a^2 e^{-at} \frac{M_0}{r} + \mathcal{O}\left(e^{-2at} \frac{M_0^2}{r^2}\right), \quad (54)$$

$$\langle T \rangle_{4D} = -\frac{n_B a^4 e^{-at} M_0}{1440\pi^2 r} + \mathcal{O}\left(e^{-2at} \frac{M_0^2}{r^2}\right), \quad (55)$$

and Eq. (51) is thus solved to leading order at large r for

$$a = \frac{\sqrt{2880}\pi}{\sqrt{n_B} \ell_p}. \quad (56)$$

The CFT trace anomaly in this case is subleading,

$$\langle T \rangle_{\text{CFT}} \approx \frac{a^4 e^{-2at} M_0^2}{6\ell_p^2 \sigma^2 r^2}, \quad (57)$$

and one then concludes that the AdS-CFT correspondence is wildly violated. As we mentioned at the end of Sec. III B 2, this is not necessarily a flaw.

Finally, the nonvanishing components of the energy-momentum tensor, again to leading order at large r , are given by

$$T^t_r \simeq -T^r_t \simeq \frac{2ae^{-at}M}{\ell_p^2 r^2},$$

$$T^\theta_\theta \simeq T^\phi_\phi \simeq \frac{a^3 e^{-at}M}{\ell_p^2 r}, \quad (58)$$

and the luminosity is

$$\dot{M} \sim -aM_0 e^{-at}. \quad (59)$$

In order to fix a reasonable value for M_0 , we can now assume that, for sufficiently large ADM mass, the standard Hawking relation holds [8],

$$\dot{M} \sim -n_B K \frac{\ell_p^2}{M^2}, \quad (60)$$

where K is the same coefficient which appears in Eq. (16). The transition to the new regime would then occur when the two expressions of the luminosity, Eqs. (59) and (60), match (at $t=0$), that is for

$$M_0 \sim \left(n_B \frac{\ell_p^2}{a} \right)^{1/3} \simeq 0.1 n_B^{1/2} \ell_p$$

$$\simeq 0.1 \sigma^{-1} \simeq 0.1 \text{ mm}, \quad (61)$$

where we have estimated n_B as in Eq. (2) in the second line. Since $\sigma M_0 < 1$, it is no more a surprise that the holographic description fails for the present case. Note that the luminosity (59) vanishes for $M=0$ [whereas the expression in Eq. (60) diverges], that is the temperature of such black holes is much lower than the canonical one. This is just the kind of improvement one expects from energy conservation and the use of the microcanonical picture for the system consisting of the black hole and its Hawking radiation [23] near the end-point of the evaporation [24,46].

Of course, the above calculations are just suggestive of how to tackle the problem, and are not meant to be conclusive. One point is however clear, that in a brane-world scenario one has to accommodate just for the one vacuum condition in Eq. (51), which is therefore easier to approach than the four-dimensional analogue. The hard part of the task is then moved to the bulk: the brane metric we have considered must not give rise to spurious singularities off the brane. If the evaporation is complete, this is obviously true for the Schwarzschild metric and Eq. (53) in the limit $t \rightarrow \infty$, but a

complete analysis of the bulk equations for such a time-dependent brane metric is intractable analytically.

Let us finally speculate on the basis that a vanishing four-dimensional ADM mass is not equivalent to zero proper mass, since terms of higher order in $1/r$, such as those considered in Eq. (41), may survive after the time when M has vanished (or, rather, approached the critical value ℓ_g). They are in general expected to appear as generated by the nonvanishing E_{ij} and the trace anomaly (A1d) they give rise to is smaller for larger n (and the same value of the “mass” parameter $M_{(n)}$). This opens up a wealth of new possibilities for the brane-world. Since we have shown evidence that the late stage of the evaporation is likely a (slow) exponential decay, one can capture an instantaneous picture of its evolution in time [i.e., apply the adiabatic approximation in order to obtain the static form (13)] and expand that in powers of $1/r$,

$$N = 1 - \sum_{n \geq \bar{n}} \left(\frac{M_{(n)}}{r} \right)^n, \quad (62)$$

where $M_{(1)} \equiv 2M$ and $n = \bar{n}$ is the smallest order for which the coefficient $M_{(n)} \neq 0$, so that, although the black hole remains five dimensional, its profile looks like it is higher dimensional. Then, one would also have a “remnant” trace anomaly which is approximately given by the expression in Eq. (A1d) with $n = \bar{n}$. If \bar{n} increases in time, the corresponding trace anomaly decreases in time and the black hole appears as a higher and higher dimensional object from the point of view of an observer restricted on the four-dimensional brane-world. Correspondingly, the space-time (brane) around the singularity looks flatter and flatter [see the Ricci tensor elements in Eq. (42)]. More precisely, once the horizon radius has approached ℓ_g , a geometric description of the space surrounding the central singularity becomes questionable, and just the large r limit of the metric can be given sense. The latter is practically flat for $r > M_{(\bar{n})}$ when $\bar{n} \geq 2$.

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APPENDIX A: TRACE ANOMALIES

We display here the complete expressions of the trace anomalies for the static cases I, II, and III of Sec. III, and $n_B = 1$. In obvious notation:

$$I\langle T \rangle_{4D} = \frac{\left(1 - \frac{3M}{2r}\right)^{-4} M^2}{25920 \pi^2 r^6} \left[18(24 + 8\eta + \eta^2) - 16(162 + 81\eta + 10\eta^2) \frac{M}{r} \right.$$

$$\left. + 12(486 + 315\eta + 49\eta^2) \frac{M^2}{r^2} + 216(27 + 21\eta + 4\eta^2) \frac{M^3}{r^3} + 9(243 + 216\eta + 48\eta^2) \frac{M^4}{r^4} \right], \quad (A1a)$$

$$\begin{aligned} \text{II}\langle T \rangle_{4D} = & \frac{(1+\eta)^2 M^2}{240\pi^2 r^6} \left(\eta + \sqrt{1 - \frac{2M}{r}(1+\eta)} \right)^{-4} \left[4 + \frac{3}{2} \eta^2 (9 + \eta^2) + \eta(12 + 7\eta) \sqrt{1 - \frac{2M}{r}(1+\eta)} \right. \\ & \left. - (1+\eta) \left(16 + 27\eta^2 + 24\eta \sqrt{1 - \frac{2M}{r}(1+\eta)} \right) \frac{M}{r} + 16(1+\eta)^2 \frac{M^2}{r^2} \right], \end{aligned} \quad (\text{A1b})$$

$$\text{III}\langle T \rangle_{4D} = \frac{M^2}{60\pi^2 r^6} \left(1 - \eta \frac{M}{r} + \frac{13\eta^2 M^2}{48r^2} \right), \quad (\text{A1c})$$

$$\begin{aligned} {}^{(n)}\langle T \rangle_{4D} = & \frac{M_{(n)}^n}{5760\pi^2 r^{4+n}} \left[(n^2 - 1)(n^2 - 4) \right. \\ & \left. - (3n^4 - 8n^3 - 23n^2 + 4) \frac{M_{(n)}^n}{r^n} \right]. \end{aligned} \quad (\text{A1d})$$

From the AdS-CFT correspondence [28] one instead obtains

$$\text{I}\langle T \rangle_{\text{CFT}} = \frac{\eta^2 M^2}{6\ell_p^2 \sigma^2 r^6} \left(1 - \frac{3M}{2r} \right)^{-4} \left(1 - \frac{8M}{3r} + \frac{2M^2}{r^2} \right), \quad (\text{A2a})$$

$$\text{II}\langle T \rangle_{\text{CFT}} = \frac{3\eta^2 (1-\eta)^2 M^2}{2\ell_p^2 \sigma^2 r^6} \left(\eta + \sqrt{1 - \frac{2M}{r}(1+\eta)} \right)^{-2}, \quad (\text{A2b})$$

$$\text{III}\langle T \rangle_{\text{CFT}} = \frac{\eta^2 M^2}{4\ell_p^2 \sigma^2 r^8}, \quad (\text{A2c})$$

$${}^{(n)}\langle T \rangle_{\text{CFT}} = \frac{n^2 + 8n + 4}{24} \frac{(1-n)^2 M_{(n)}^{2n}}{\ell_p^2 \sigma^2 r^{2(2+n)}}. \quad (\text{A2d})$$

APPENDIX B: SMALL BLACK HOLES

In the approach of Ref. [10] the bulk metric is taken of the form

$$ds^2 = -N(r, z) dt^2 + A(r, z) dr^2 + \rho^2(r, z) d\Omega^2 + dz^2. \quad (\text{B1})$$

For the brane metric in Eq. (41) with $n=2$, the computed

metric components (to order $1/r^6$, for the sake of simplicity) are then given by

$$\begin{aligned} N = e^{-\sigma z} & \left\{ 1 - \frac{M_{(2)}^2}{r^2} + (1 - e^{\sigma z})^2 \frac{M_{(2)}^2}{\sigma^2 r^4} \right. \\ & - [2 + \sigma^2 M_{(2)}^2 - 2e^{\sigma z}(6 + \sigma^2 M_{(2)}^2) \\ & \left. + e^{2\sigma z}(6 + \sigma^2 M_{(2)}^2 - 12\sigma z) + 4e^{3\sigma z}] \frac{M_{(2)}^2}{\sigma^4 r^6} \right\}, \end{aligned} \quad (\text{B2a})$$

$$\begin{aligned} A = e^{-\sigma z} & \left\{ 1 + \frac{M_{(2)}^2}{r^2} + [\sigma^2 M_{(2)}^2 + (1 - e^{\sigma z})^2] \frac{M_{(2)}^2}{\sigma^2 r^4} \right. \\ & - \{3(1 + e^{\sigma z})^2 \sigma^2 M_{(2)}^2 - 3\sigma^4 M_{(2)}^4 \\ & \left. + 2[1 - 6e^{\sigma z} + 3e^{2\sigma z}(1 - 2\sigma z) + 2e^{3\sigma z}]\} \frac{M_{(2)}^2}{3\sigma^4 r^6} \right\}, \end{aligned} \quad (\text{B2b})$$

$$\begin{aligned} \rho^2 = r^2 e^{-\sigma z} & \left\{ 1 - (1 - e^{\sigma z})^2 \frac{M_{(2)}^2}{\sigma^2 r^4} \right. \\ & \left. + [1 - 6e^{\sigma z} + 3e^{2\sigma z}(1 - 2\sigma z) + 2e^{3\sigma z}] \frac{4M_{(2)}^2}{3\sigma^4 r^6} \right\}. \end{aligned} \quad (\text{B2c})$$

Note that, as we commented upon in Ref. [10], the function ρ^2 vanishes for a finite value of z , with r held fixed, and that locates the axis of cylindrical symmetry in the Gaussian normal reference frame. The latter covers the whole bulk manifold of such black holes, since no crossing occurs between lines of different constant r (see Fig. 2).

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